An Implementation of a Positive Operator Valued Measure

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An implementation of the positive operator valued measure (POVM) is given. By using this POVM one can realize the probabilistic teleportation of an unknown two-particle state.

As a special case of the general measurement formalism, positive operator valued measure (POVM) provides the simplest means by which one can study general measurement statistics, without the necessity for knowing the post-measurement state. They are viewed as a mathematical convenience that sometimes gives extra insight into quantum measurements [1]. Some of the time, one can perform a POVM to distinguish the states, but never make an error of mis-identification. Of course this infallibility comes at the price that sometimes one obtains no information about the identity of the state [1].

Mor and Horodecki [2] used POVM to the problems of teleportation, which has been studied widely [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25] and has a number of useful applications in quantum information and quantum computation [26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44]. It was shown that a perfect conclusive teleportation can be obtained with any pure entangled states. Recently, we proposed a scheme [25] for probabilistic teleportation of an unknown two-particle state with a four-particle pure entangled state and the POVM. In Ref. [25] we only gave the POVM operators, but did not present the concrete implementation of the POVM. In this Letter we will study this problem in details, especially an implementation of the POVM will be presented.

The optimal POVM chosen by us in Ref. [25] contains five elements (here we use a simple form, there is only a little difference between them)

$$P_i = q^2|\Psi_i\rangle\langle\Psi_i|;\ \ (i=1,2,3,4);\ \ P_5 = I - q^2\Sigma_{i=1}^4|\Psi_i\rangle\langle\Psi_i|, \tag{1}$$

where

$$\begin{aligned}
|\Psi_1\rangle &= \tfrac{1}{\alpha}|00\rangle+\tfrac{1}{\beta}|01\rangle+\tfrac{1}{\gamma}|10\rangle+\tfrac{1}{\delta}|11\rangle,\ \ |\Psi_2\rangle = \tfrac{1}{\alpha}|00\rangle+\tfrac{1}{\beta}|01\rangle-\tfrac{1}{\gamma}|10\rangle-\tfrac{1}{\delta}|11\rangle,\\
|\Psi_3\rangle &= \tfrac{1}{\alpha}|00\rangle-\tfrac{1}{\beta}|01\rangle+\tfrac{1}{\gamma}|10\rangle-\tfrac{1}{\delta}|11\rangle,\ \ |\Psi_4\rangle = \tfrac{1}{\alpha}|00\rangle-\tfrac{1}{\beta}|01\rangle-\tfrac{1}{\gamma}|10\rangle+\tfrac{1}{\delta}|11\rangle,
\end{aligned} \tag{2}$$

I is an identity operator, $\alpha,\beta,\gamma,\delta$ are nonzero real numbers with $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}+\frac{1}{\delta^2}=1$, q is a coefficient relating to α,β,γ, and δ such that $1\leq\frac{1}{q^2}\leq 4$, and makes P_5 to be a positive operator. Of course if we choose $q^2=\frac{\mu^2}{4}$, where μ^2 is the smallest one in the set $\{\alpha^2,\beta^2,\gamma^2,\delta^2\}$, we can obtain the highest probability of successful teleportation of an unknown two-particle state [25].

In order to implement this POVM, we construct a unitary operation U, which satisfies

$$U|\psi\rangle|000\rangle = \sqrt{P_1}|\psi\rangle|000\rangle+\sqrt{P_2}|\psi\rangle|001\rangle+\sqrt{P_3}|\psi\rangle|010\rangle+\sqrt{P_4}|\psi\rangle|011\rangle+\sqrt{P_5}|\psi\rangle|100\rangle \tag{3}$$

for an arbitrary two-particle state $|\psi\rangle$. Here the last three qubits are the auxiliary particles. When the unitary operator U has been performed, we can measure the auxiliary particles and obtain the measurement result. It is easy to see that if U is a unitary operator such that

$$\begin{aligned}
U|00\rangle|000\rangle &= \frac{q}{\alpha}(\frac{1}{\alpha}|00\rangle+\frac{1}{\beta}|01\rangle+\frac{1}{\gamma}|10\rangle+\frac{1}{\delta}|11\rangle)|000\rangle+\frac{q}{\alpha}(\frac{1}{\alpha}|00\rangle+\frac{1}{\beta}|01\rangle-\frac{1}{\gamma}|10\rangle-\frac{1}{\delta}|11\rangle)|001\rangle\\
&+\frac{q}{\alpha}(\frac{1}{\alpha}|00\rangle-\frac{1}{\beta}|01\rangle+\frac{1}{\gamma}|10\rangle-\frac{1}{\delta}|11\rangle)|010\rangle+\frac{q}{\alpha}(\frac{1}{\alpha}|00\rangle-\frac{1}{\beta}|01\rangle-\frac{1}{\gamma}|10\rangle+\frac{1}{\delta}|11\rangle)|011\rangle+u|00\rangle|100\rangle,
\end{aligned} \tag{4}$$

$$\begin{aligned}
U|01\rangle|000\rangle &= \frac{q}{\beta}(\frac{1}{\alpha}|00\rangle+\frac{1}{\beta}|01\rangle+\frac{1}{\gamma}|10\rangle+\frac{1}{\delta}|11\rangle)|000\rangle+\frac{q}{\beta}(\frac{1}{\alpha}|00\rangle+\frac{1}{\beta}|01\rangle-\frac{1}{\gamma}|10\rangle-\frac{1}{\delta}|11\rangle)|001\rangle\\
&-\frac{q}{\beta}(\frac{1}{\alpha}|00\rangle-\frac{1}{\beta}|01\rangle+\frac{1}{\gamma}|10\rangle-\frac{1}{\delta}|11\rangle)|010\rangle-\frac{q}{\beta}(\frac{1}{\alpha}|00\rangle-\frac{1}{\beta}|01\rangle-\frac{1}{\gamma}|10\rangle+\frac{1}{\delta}|11\rangle)|011\rangle+v|01\rangle|100\rangle,
\end{aligned} \tag{5}$$

$$
\begin{aligned}
U|10\rangle|000\rangle \quad &= \frac{q}{\gamma}(\frac{1}{\alpha}|00\rangle+\frac{1}{\beta}|01\rangle+\frac{1}{\gamma}|10\rangle+\frac{1}{\delta}|11\rangle)|000\rangle-\frac{q}{\gamma}(\frac{1}{\alpha}|00\rangle+\frac{1}{\beta}|01\rangle-\frac{1}{\gamma}|10\rangle-\frac{1}{\delta}|11\rangle)|001\rangle \\
&+\frac{q}{\gamma}(\frac{1}{\alpha}|00\rangle-\frac{1}{\beta}|01\rangle+\frac{1}{\gamma}|10\rangle-\frac{1}{\delta}|11\rangle)|010\rangle-\frac{q}{\gamma}(\frac{1}{\alpha}|00\rangle-\frac{1}{\beta}|01\rangle-\frac{1}{\gamma}|10\rangle+\frac{1}{\delta}|11\rangle)|011\rangle+w|10\rangle|100\rangle,
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
U|11\rangle|000\rangle \quad &= \frac{q}{\delta}(\frac{1}{\alpha}|00\rangle+\frac{1}{\beta}|01\rangle+\frac{1}{\gamma}|10\rangle+\frac{1}{\delta}|11\rangle)|000\rangle-\frac{q}{\delta}(\frac{1}{\alpha}|00\rangle+\frac{1}{\beta}|01\rangle-\frac{1}{\gamma}|10\rangle-\frac{1}{\delta}|11\rangle)|001\rangle \\
&-\frac{q}{\delta}(\frac{1}{\alpha}|00\rangle-\frac{1}{\beta}|01\rangle+\frac{1}{\gamma}|10\rangle-\frac{1}{\delta}|11\rangle)|010\rangle+\frac{q}{\delta}(\frac{1}{\alpha}|00\rangle-\frac{1}{\beta}|01\rangle-\frac{1}{\gamma}|10\rangle+\frac{1}{\delta}|11\rangle)|011\rangle+p|11\rangle|100\rangle,
\end{aligned}
\tag{7}
$$

then Eq. (3) holds. Here

$$u=\sqrt{1-\tfrac{4q^2}{\alpha^2}},\;\; v=\sqrt{1-\tfrac{4q^2}{\beta^2}},\;\; w=\sqrt{1-\tfrac{4q^2}{\gamma^2}},\;\; p=\sqrt{1-\tfrac{4q^2}{\delta^2}}. \tag{8}$$

For the sake of convenience we introduce the following parameters

$$s=\sqrt{\alpha^2+\beta^2},\;\; y=\sqrt{\alpha^2+\delta^2},\;\; z=\sqrt{\beta^2+\gamma^2},\;\; t=\sqrt{\gamma^2+\delta^2}. \tag{9}$$

A little thought shows that the following unitary operator

$$
U=
\left(\begin{array}{cccccccccccccccccccccccc}
\frac{q}{\alpha^2} & \frac{\alpha}{2y} & \frac{\alpha}{2s} & \frac{-\alpha}{2y} & \frac{u}{2\alpha} & & \frac{q}{\alpha\beta} & \frac{\beta t}{2\gamma\delta s} & 0 & \frac{-\beta t}{2\gamma\delta s} & \frac{v}{2\alpha} & & \frac{q}{\alpha\gamma} & \frac{\delta z}{2\beta\gamma y} & 0 & \frac{-\delta z}{2\beta\gamma y} & 0 & & \frac{q}{\alpha\delta} & \frac{w}{2\alpha} & 0 & \frac{\alpha}{2s} & \frac{p}{2\alpha} & \\
\frac{q}{\alpha^2} & \frac{\alpha}{2y} & \frac{-\alpha}{2s} & \frac{\alpha}{2y} & \frac{u}{2\alpha} & & \frac{q}{\alpha\beta} & \frac{\beta t}{2\gamma\delta s} & 0 & \frac{\beta t}{2\gamma\delta s} & \frac{v}{2\alpha} & & \frac{-q}{\alpha\gamma} & \frac{-\delta z}{2\beta\gamma y} & 0 & \frac{-\delta z}{2\beta\gamma y} & 0 & & \frac{-q}{\alpha\delta} & \frac{-w}{2\alpha} & 0 & \frac{\alpha}{2s} & \frac{-p}{2\alpha} & \\
\frac{q}{\alpha^2} & \frac{-\alpha}{2y} & \frac{\alpha}{2s} & \frac{\alpha}{2y} & \frac{u}{2\alpha} & & \frac{-q}{\alpha\beta} & \frac{\beta t}{2\gamma\delta s} & 0 & \frac{-\beta t}{2\gamma\delta s} & \frac{-v}{2\alpha} & & \frac{q}{\alpha\gamma} & \frac{-\delta z}{2\beta\gamma y} & 0 & \frac{\delta z}{2\beta\gamma y} & 0 & & \frac{-q}{\alpha\delta} & \frac{w}{2\alpha} & 0 & \frac{\alpha}{2s} & \frac{-p}{2\alpha} & \\
\frac{q}{\alpha^2} & \frac{-\alpha}{2y} & \frac{-\alpha}{2s} & \frac{-\alpha}{2y} & \frac{u}{2\alpha} & & \frac{-q}{\alpha\beta} & \frac{\beta t}{2\gamma\delta s} & 0 & \frac{\beta t}{2\gamma\delta s} & \frac{-v}{2\alpha} & & \frac{-q}{\alpha\gamma} & \frac{\delta z}{2\beta\gamma y} & 0 & \frac{\delta z}{2\beta\gamma y} & 0 & & \frac{q}{\alpha\delta} & \frac{-w}{2\alpha} & 0 & \frac{\alpha}{2s} & \frac{p}{2\alpha} & \\
u & 0 & 0 & 0 & \frac{2q}{-\alpha} & & 0 & 0 & 0 & 0 & 0 & & 0 & 0 & 0 & 0 & 0 & & 0 & 0 & 0 & 0 & 0 & \\
 & & & & & I_3 & & & & & & & & & & & & & & & & & & \\
\frac{q}{\alpha\beta} & 0 & \frac{-\beta}{2s} & 0 & \frac{u}{2\beta} & & \frac{q}{\beta^2} & \frac{\alpha t}{2\gamma\delta s} & \frac{\beta}{2z} & \frac{-\alpha t}{2\gamma\delta s} & \frac{v}{2\beta} & & \frac{q}{\beta\gamma} & \frac{-\gamma y}{2\alpha\delta z} & 0 & \frac{\gamma y}{2\alpha\delta z} & 0 & & \frac{q}{\beta\delta} & \frac{w}{2\beta} & \frac{\beta}{2z} & \frac{-\beta}{2s} & \frac{p}{2\beta} & \\
\frac{q}{\alpha\beta} & 0 & \frac{\beta}{2s} & 0 & \frac{u}{2\beta} & & \frac{q}{\beta^2} & \frac{\alpha t}{2\gamma\delta s} & \frac{-\beta}{2z} & \frac{\alpha t}{2\gamma\delta s} & \frac{v}{2\beta} & & \frac{-q}{\beta\gamma} & \frac{\gamma y}{2\alpha\delta z} & 0 & \frac{\gamma y}{2\alpha\delta z} & 0 & & \frac{-q}{\beta\delta} & \frac{-w}{2\beta} & \frac{\beta}{2z} & \frac{-\beta}{2s} & \frac{-p}{2\beta} & \\
\frac{-q}{\alpha\beta} & 0 & \frac{\beta}{2s} & 0 & \frac{-u}{2\beta} & & \frac{q}{\beta^2} & \frac{-\alpha t}{2\gamma\delta s} & \frac{\beta}{2z} & \frac{\alpha t}{2\gamma\delta s} & \frac{v}{2\beta} & & \frac{-q}{\beta\gamma} & \frac{-\gamma y}{2\alpha\delta z} & 0 & \frac{\gamma y}{2\alpha\delta z} & 0 & & \frac{q}{\beta\delta} & \frac{-w}{2\beta} & \frac{\beta}{2z} & \frac{\beta}{2s} & \frac{p}{2\beta} & \\
\frac{-q}{\alpha\beta} & 0 & \frac{-\beta}{2s} & 0 & \frac{-u}{2\beta} & & \frac{q}{\beta^2} & \frac{-\alpha t}{2\gamma\delta s} & \frac{-\beta}{2z} & \frac{-\alpha t}{2\gamma\delta s} & \frac{v}{2\beta} & & \frac{q}{\beta\gamma} & \frac{\gamma y}{2\alpha\delta z} & 0 & \frac{\gamma y}{2\alpha\delta z} & 0 & & \frac{-q}{\beta\delta} & \frac{w}{2\beta} & \frac{\beta}{2z} & \frac{\beta}{2s} & \frac{-p}{2\beta} & \\
0 & 0 & 0 & 0 & 0 & & v & 0 & 0 & 0 & \frac{2q}{-\beta} & & 0 & 0 & 0 & 0 & 0 & & 0 & 0 & 0 & 0 & 0 & \\
 & & & & & & & & & & & I_3 & & & & & & & & & & & & \\
\frac{q}{\alpha\gamma} & 0 & 0 & 0 & \frac{u}{2\gamma} & & \frac{q}{\beta\gamma} & \frac{-\delta s}{2\alpha\beta t} & \frac{-\gamma}{2z} & \frac{\delta s}{2\alpha\beta t} & \frac{v}{2\gamma} & & \frac{q}{\gamma^2} & \frac{-\beta y}{2\alpha\delta z} & \frac{\gamma}{2t} & \frac{\beta y}{2\alpha\delta z} & \frac{\gamma}{2t} & & \frac{q}{\gamma\delta} & \frac{w}{2\gamma} & \frac{-\gamma}{2z} & 0 & \frac{p}{2\gamma} & \\
\frac{-q}{\alpha\gamma} & 0 & 0 & 0 & \frac{-u}{2\gamma} & & \frac{-q}{\beta\gamma} & \frac{\delta s}{2\alpha\beta t} & \frac{-\gamma}{2z} & \frac{\delta s}{2\alpha\beta t} & \frac{-v}{2\gamma} & & \frac{q}{\gamma^2} & \frac{-\beta y}{2\alpha\delta z} & \frac{-\gamma}{2t} & \frac{-\beta y}{2\alpha\delta z} & \frac{\gamma}{2t} & & \frac{q}{\gamma\delta} & \frac{w}{2\gamma} & \frac{\gamma}{2z} & 0 & \frac{p}{2\gamma} & \\
\frac{q}{\alpha\gamma} & 0 & 0 & 0 & \frac{u}{2\gamma} & & \frac{-q}{\beta\gamma} & \frac{-\delta s}{2\alpha\beta t} & \frac{\gamma}{2z} & \frac{\delta s}{2\alpha\beta t} & \frac{-v}{2\gamma} & & \frac{q}{\gamma^2} & \frac{\beta y}{2\alpha\delta z} & \frac{\gamma}{2t} & \frac{-\beta y}{2\alpha\delta z} & \frac{\gamma}{2t} & & \frac{-q}{\gamma\delta} & \frac{w}{2\gamma} & \frac{\gamma}{2z} & 0 & \frac{-p}{2\gamma} & \\
\frac{-q}{\alpha\gamma} & 0 & 0 & 0 & \frac{-u}{2\gamma} & & \frac{q}{\beta\gamma} & \frac{\delta s}{2\alpha\beta t} & \frac{\gamma}{2z} & \frac{\delta s}{2\alpha\beta t} & \frac{v}{2\gamma} & & \frac{q}{\gamma^2} & \frac{\beta y}{2\alpha\delta z} & \frac{-\gamma}{2t} & \frac{\beta y}{2\alpha\delta z} & \frac{\gamma}{2t} & & \frac{-q}{\gamma\delta} & \frac{w}{2\gamma} & \frac{-\gamma}{2z} & 0 & \frac{-p}{2\gamma} & \\
0 & 0 & 0 & 0 & 0 & & 0 & 0 & 0 & 0 & 0 & & w & 0 & 0 & 0 & 0 & & 0 & \frac{2q}{-\gamma} & 0 & 0 & 0 & \\
 & & & & & & & & & & & & & & & & & I_3 & & & & & & \\
\frac{q}{\alpha\delta} & \frac{-\delta}{2y} & 0 & \frac{\delta}{2y} & \frac{u}{2\delta} & & \frac{q}{\beta\delta} & \frac{-\gamma s}{2\alpha\beta t} & 0 & \frac{\gamma s}{2\alpha\beta t} & \frac{v}{2\delta} & & \frac{q}{\gamma\delta} & \frac{\alpha z}{2\beta\gamma y} & \frac{-\delta}{2t} & \frac{-\alpha z}{2\beta\gamma y} & \frac{-\delta}{2t} & & \frac{q}{\delta^2} & \frac{w}{2\delta} & 0 & 0 & \frac{p}{2\delta} & \\
\frac{-q}{\alpha\delta} & \frac{\delta}{2y} & 0 & \frac{\delta}{2y} & \frac{-u}{2\delta} & & \frac{-q}{\beta\delta} & \frac{\gamma s}{2\alpha\beta t} & 0 & \frac{\gamma s}{2\alpha\beta t} & \frac{-v}{2\delta} & & \frac{q}{\gamma\delta} & \frac{\alpha z}{2\beta\gamma y} & \frac{\delta}{2t} & \frac{\alpha z}{2\beta\gamma y} & \frac{-\delta}{2t} & & \frac{q}{\delta^2} & \frac{w}{2\delta} & 0 & 0 & \frac{p}{2\delta} & \\
\frac{-q}{\alpha\delta} & \frac{-\delta}{2y} & 0 & \frac{\delta}{2y} & \frac{-u}{2\delta} & & \frac{q}{\beta\delta} & \frac{\gamma s}{2\alpha\beta t} & 0 & \frac{-\gamma s}{2\alpha\beta t} & \frac{v}{2\delta} & & \frac{-q}{\gamma\delta} & \frac{\alpha z}{2\beta\gamma y} & \frac{\delta}{2t} & \frac{-\alpha z}{2\beta\gamma y} & \frac{\delta}{2t} & & \frac{q}{\delta^2} & \frac{-w}{2\delta} & 0 & 0 & \frac{p}{2\delta} & \\
\frac{q}{\alpha\delta} & \frac{\delta}{2y} & 0 & \frac{\delta}{2y} & \frac{u}{2\delta} & & \frac{-q}{\beta\delta} & \frac{-\gamma s}{2\alpha\beta t} & 0 & \frac{-\gamma s}{2\alpha\beta t} & \frac{-v}{2\delta} & & \frac{-q}{\gamma\delta} & \frac{\alpha z}{2\beta\gamma y} & \frac{-\delta}{2t} & \frac{\alpha z}{2\beta\gamma y} & \frac{\delta}{2t} & & \frac{q}{\delta^2} & \frac{-w}{2\delta} & 0 & 0 & \frac{p}{2\delta} & \\
0 & 0 & 0 & 0 & 0 & & 0 & 0 & 0 & 0 & 0 & & 0 & 0 & 0 & 0 & 0 & & p & 0 & 0 & 0 & \frac{2q}{-\delta} & \\
 & & & & & & & & & & & & & & & & & & & & & & & I_3
\end{array}\right)
\tag{10}
$$

can satisfy Eq.(4)-(7). Here I_3 is a 3×3 identity matrix.

We can prove that U can be expressed as

$$
\begin{aligned}
U = \quad & M \begin{pmatrix} \frac{\alpha}{s} & \frac{-\beta}{s} \\ \frac{-\beta}{s} & \frac{-\alpha}{s} \end{pmatrix}_{1,10} \begin{pmatrix} \frac{\alpha}{y} & \frac{-\delta}{y} \\ \frac{-\delta}{y} & \frac{-\alpha}{y} \end{pmatrix}_{2,27} \begin{pmatrix} \frac{\alpha}{s} & \frac{\beta}{s} \\ \frac{\beta}{s} & \frac{-\alpha}{s} \end{pmatrix}_{3,12} \begin{pmatrix} \frac{\alpha}{y} & \frac{\delta}{y} \\ \frac{\delta}{y} & \frac{-\alpha}{y} \end{pmatrix}_{4,25} \\
& \begin{pmatrix} \frac{\beta}{z} & \frac{\gamma}{z} \\ \frac{\gamma}{z} & \frac{-\beta}{z} \end{pmatrix}_{9,20} \begin{pmatrix} \frac{\beta}{z} & \frac{-\gamma}{z} \\ \frac{-\gamma}{z} & \frac{-\beta}{z} \end{pmatrix}_{11,18} \begin{pmatrix} \frac{\gamma}{t} & \frac{-\delta}{t} \\ \frac{-\delta}{t} & \frac{-\gamma}{t} \end{pmatrix}_{17,26} \begin{pmatrix} \frac{\gamma}{t} & \frac{\delta}{t} \\ \frac{\delta}{t} & \frac{-\gamma}{t} \end{pmatrix}_{19,28} \\
& \begin{pmatrix} \frac{-t}{\gamma\delta} & \frac{-s}{\alpha\beta} \\ \frac{-s}{\alpha\beta} & \frac{t}{\gamma\delta} \end{pmatrix}_{10,28} \begin{pmatrix} \frac{-t}{\gamma\delta} & \frac{-s}{\alpha\beta} \\ \frac{-s}{\alpha\beta} & \frac{t}{\gamma\delta} \end{pmatrix}_{12,26} \begin{pmatrix} \frac{y}{\alpha\delta} & \frac{-z}{\beta\gamma} \\ \frac{-z}{\beta\gamma} & \frac{-y}{\alpha\delta} \end{pmatrix}_{18,25} \begin{pmatrix} \frac{y}{\alpha\delta} & \frac{z}{\beta\gamma} \\ \frac{z}{\beta\gamma} & \frac{-y}{\alpha\delta} \end{pmatrix}_{20,27} \\
& \begin{pmatrix} \frac{-2q}{\alpha} & u \\ u & \frac{2q}{\alpha} \end{pmatrix}_{5,28} \begin{pmatrix} \frac{-2q}{\beta} & v \\ v & \frac{2q}{\beta} \end{pmatrix}_{13,27} \begin{pmatrix} w & \frac{-2q}{\gamma} \\ \frac{-2q}{\gamma} & -w \end{pmatrix}_{21,26} \begin{pmatrix} \frac{2q}{\delta} & p \\ p & \frac{-2q}{\delta} \end{pmatrix}_{25,29} \\
& \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{1,28} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{9,27} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{17,21},
\end{aligned}
\tag{11}
$$

where

$$
\begin{aligned}
M = \quad & \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix}_{1,2} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & -\sqrt{\frac{2}{3}} \end{pmatrix}_{1,3} \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}_{1,4} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{2,4} \begin{pmatrix} \sqrt{\frac{1}{3}} & \frac{\sqrt{2}}{\sqrt{3}} \\ \frac{\sqrt{2}}{\sqrt{3}} & -\sqrt{\frac{1}{3}} \end{pmatrix}_{2,3} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}_{3,4} \\
& \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix}_{9,10} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & -\sqrt{\frac{2}{3}} \end{pmatrix}_{9,11} \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}_{9,12} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{10,12} \begin{pmatrix} \sqrt{\frac{1}{3}} & \frac{\sqrt{2}}{\sqrt{3}} \\ \frac{\sqrt{2}}{\sqrt{3}} & -\sqrt{\frac{1}{3}} \end{pmatrix}_{10,11} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}_{11,12} \\
& \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix}_{17,18} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & -\sqrt{\frac{2}{3}} \end{pmatrix}_{17,19} \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}_{17,20} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{18,20} \begin{pmatrix} \sqrt{\frac{1}{3}} & \frac{\sqrt{2}}{\sqrt{3}} \\ \frac{\sqrt{2}}{\sqrt{3}} & -\sqrt{\frac{1}{3}} \end{pmatrix}_{18,19} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}_{19,20} \\
& \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix}_{25,26} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & -\sqrt{\frac{2}{3}} \end{pmatrix}_{25,27} \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}_{25,28} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{26,28} \begin{pmatrix} \sqrt{\frac{1}{3}} & \frac{\sqrt{2}}{\sqrt{3}} \\ \frac{\sqrt{2}}{\sqrt{3}} & -\sqrt{\frac{1}{3}} \end{pmatrix}_{26,27} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}_{27,28}.
\end{aligned}
\tag{12}
$$

Here $\Omega = \begin{pmatrix} \xi & \zeta \\ \zeta & -\xi \end{pmatrix}_{i,j}$ denotes a 32×32 unitary matrix or a two-level unitary matrix, where the matrix elements $\Omega_{i,i} = \xi, \Omega_{i,j} = \Omega_{j,i} = \zeta, \Omega_{j,j} = -\xi$, $\Omega_{l,l} = 1$, for $l \neq i, j$, and the other elements of matrix Ω are zero.

By making use of Gray codes one can construct a circuit implementing a two-level unitary operator Ω, where the circuit only consists of a number of controlled operations [1].

Barenco et al exhibited a general simulation of controlled operations using only one-bit gates and the two-bit controlled-not (CNOT) gates [45]. Combining the results obtained by Barenco et al. and the above decomposition of U we can give the explicit construction of the unitary operation U using one-qubit gates and two-qubit CNOT gates. For save space, we do not give an implementation of U in terms of one and two qubit operations and also do not depict out the quantum circuit illustrating the procedure of the implementation of the POVM.

In summary, an implementation of a POVM has been given. By using this POVM one can realize the probabilistic teleportation of an unknown two-particle state. We hope that this POVM will be realized by experiment, furthermore one will really see the probabilistic teleportation of an unknown two-particle state with a four-particle pure entangled state and POVM.

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